

## TURBULENCE

# Synthetic dissipation and cascade fluxes in a turbulent quantum gas

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Scale-invariant fluxes are the defining property of turbulent cascades, but their direct measurement is a challenging experimental problem. Here we perform such a measurement for a direct energy cascade in a turbulent quantum gas. Using a time-periodic force, we inject energy at a large length scale and generate a cascade in a uniformly trapped three-dimensional Bose gas. The adjustable trap depth provides a high-momentum cutoff  $k_D$ , which realizes a synthetic dissipation scale. This gives us direct access to the particle flux across a momentum shell of radius  $k_D$ , and the tunability of  $k_D$  allows for a clear demonstration of the zeroth law of turbulence. Moreover, our time-resolved measurements give unique access to the pre-steady-state dynamics, when the cascade front propagates in momentum space.

The discovery of a universal law describing the transfer of energy from large to small length scales in turbulent flows was a conceptual breakthrough (1, 2). Despite their complex spatiotemporal dynamics, turbulent flows often obey a simple generic picture: The energy injected into the system at a large length scale is gradually transferred to ever smaller ones, flowing locally in Fourier space through the so-called inertial range where no dissipation occurs, until it is dissipated at some small length scale. In Fig. 1A, we depict such turbulent-cascade dynamics for a compressible field in real space. The field is initially at rest. At times  $t > 0$ , an external force creates excitations at a large length scale  $1/k_F$ . These excitations propagate to smaller length scales owing to their nonlinear interactions. Once they first reach the dissipation scale  $1/k_D$ , at time  $t_d$ , the field fluctuates on all length scales from  $1/k_F$  to  $1/k_D$ . If a steady state is established within the momentum range  $k_F$  to  $k_D$ , from thereon energy is dissipated at  $k_D$  at the same rate at which it is injected at  $k_F$ . In such a steady state, the momentum-space distributions of quantities such as the energy or wave amplitude are generically scale-free power laws.

Many quantitative theoretical predictions about turbulence are based on taking the mathematical limits  $k_F \rightarrow 0$  and  $k_D \rightarrow \infty$  (3).

Such formal treatments lead to predictions that are elegant but often counterintuitive. A key prediction of this kind is that for  $k_D \rightarrow \infty$ , the steady-state cascade corresponds to a scale-invariant ( $k$ -independent) energy flux through momentum space, but no particle flux (4).

Experimentally, the steady-state power-law spectra of various quantities have been extensively studied (5–9), whereas the equally fundamental cascade fluxes are harder to measure (10–13). Recently, ultracold atomic gases have emerged as a versatile platform for studies of turbulence (9, 14–22), offering experimental possibilities unavailable in other systems. Here, we use an atomic gas to directly measure cascade fluxes in a turbulent system. Our dissipation scale is tuneable, which allows us to explore how the fluxes depend on  $k_D$ , and to reconcile the experimental observations with the formal predictions for  $k_D \rightarrow \infty$ . Our system also allows a time-resolved study of the initial stage of turbulence (23–25), when a steady state is not yet established, which reveals how the cascade front propagates in momentum space.

Our experiment starts with a weakly interacting Bose-Einstein condensate of  $N \approx 1.2 \times 10^5$  atoms of  $^{87}\text{Rb}$  in the uniform potential of a cylindrical optical box trap of radius  $R \approx 16 \mu\text{m}$  and length  $L \approx 27 \mu\text{m}$  (Fig. 1B) (26). At the end of the initial preparation of the gas, the noncondensed fraction is  $<10\%$  and the chemical potential is  $\mu \approx k_B \times 2 \text{ nK}$ , corresponding to a healing length  $\xi \approx 1.2 \mu\text{m} \ll R, L$ . We initiate a turbulent cascade by injecting energy at the system-size length scale (corresponding to a small momentum  $k_F$ ), using a spatially uniform force  $\mathbf{F}_s(\mathbf{r}, t) = F_0 \sin(\omega_s t) \hat{\mathbf{x}}$ , where  $\hat{\mathbf{x}}$  is a unit vector along the box symmetry axis,  $F_0 L \approx k_B \times 2.5 \text{ nK}$ , and  $\omega_s \approx 2\pi \times 9 \text{ Hz}$  is tuned to resonantly excite the sound wave of wavelength  $2L$  (so  $k_F = \pi/L$ ) (27). This anisotropic forcing of the matter-wave field is represented in Fig. 1C as a small dark blue

area elongated along  $k_x$ . As shown in (9), after several seconds of shaking, the momentum distribution of the gas in the inertial range is statistically isotropic and time-invariant,  $n(\mathbf{k}, t) \approx \langle n(k) \rangle \propto k^{-\gamma}$ , with  $\gamma \approx 3.5$  (28, 29). The time invariance implies that the energy and particle fluxes through this  $k$ -range are  $k$ -independent, but it does not reveal their values. Here, we extract the cascade fluxes by studying the dissipation in our gas.

In conventional fluids, one observes macroscopic (hydrodynamic) degrees of freedom, and the dissipation occurs in the form of heating (i.e., transfer of energy into the microscopic degrees of freedom). This dissipation is set by the viscosity  $\nu$ , which is generally not tuneable. Moreover, the resulting minute heating is often difficult to measure because of thermal coupling of the fluid with its surroundings (30). Our system is thermally isolated from the environment, and we have direct access to all the microscopic degrees of freedom, so the dissipation occurs only in the form of (readily measurable) particle loss. The optical box (Fig. 1B) has a non-infinite energy depth  $U_D$ , so particles with a sufficiently large energy leave the box; in momentum space,  $U_D$  corresponds to a sphere of radius  $k_D = \sqrt{2mU_D}/\hbar$  (Fig. 1C), where  $m$  is the atom mass and  $\hbar$  is Planck's constant divided by  $2\pi$ . This simple feature realizes a synthetic dissipation scale, with  $U_D$  defining the particle and energy sink. Crucially, this dissipation scale can be tuned by changing the trapping laser power (31).

Formally, within the assumptions of the wave-turbulence theory, the equations of motion lead to a continuity equation, with a source and a sink, that is local in momentum space (3):

$$\frac{\partial n(\mathbf{k}, t)}{\partial t} = F(\mathbf{k}, t) - D(\mathbf{k}, t) - \nabla_{\mathbf{k}} \cdot \mathbf{\Pi}_n(\mathbf{k}, t) \quad (1)$$

Here  $F(\mathbf{k}, t)$  corresponds to the external force,  $D(\mathbf{k}, t)$  describes the dissipation, and  $\nabla_{\mathbf{k}} \cdot \mathbf{\Pi}_n$  captures the nonlinear interactions, where  $\mathbf{\Pi}_n$  is the particle flux. For  $F = D = 0$ , the steady-state solutions are zero-flux equilibrium thermodynamic states. If  $F$  and  $D$  are nonzero but are localized in  $k$  space, one can also get non-equilibrium steady-state solutions with a nonzero scale-independent flux sustained by the source  $F$  and the sink  $D$ .

For an isotropic outflow, the total radial particle flux is  $\Pi_n(k) = 4\pi k^2 |\mathbf{\Pi}_n(\mathbf{k})|$ . Hence, from Eq. 1, in the inertial range  $4\pi k^2 \partial n/\partial t = -\partial \Pi_n/\partial k$ . Integrating over  $k$  shows that we can measure the particle flux through the shell at  $k_D$  by simply counting the atoms remaining in the trap (see Fig. 1C):

$$\frac{\partial N}{\partial t} \equiv -\Pi_n(k_D, t) \quad (2)$$

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For a (non-equilibrium) steady state, with time-invariant  $n(k)$  in the inertial range (9), the particle flux is  $k$ - and  $t$ -independent (32), so  $\Pi_n(k_D, t) = \Pi_n(k, t) = \Pi_n$ .

In steady state, the total radial energy flux,  $\Pi_\varepsilon(k, t)$ , is also  $k$ - and  $t$ -independent in the inertial range, and is equal to the rate of energy dissipation. To relate it to  $\Pi_n$ , we consider the pertinent case of weakly interacting particles with a dispersion relation  $\omega(k)$ , so the energy spectrum is  $\mathcal{E}(k, t) = \hbar\omega(k)n(k, t)$ ; in our case,  $\omega(k) \propto k^2$ . At  $k < k_D$ , microscopic interactions drive particles to both lower and higher  $k$ , so the relationship between the net energy and particle fluxes,  $\Pi_\varepsilon$  and  $\Pi_n$ , is non-trivial; one might naively expect that  $\Pi_\varepsilon(k) = \hbar\omega(k)\Pi_n(k)$ , but this cannot be true if both  $\Pi_\varepsilon$  and  $\Pi_n$  are  $k$ -independent and  $\omega(k)$  is not. However, at  $k_D$  the particles flow only one way because there is no “backflow” from the sink into the inertial range, so one can intuitively write

$$\Pi_\varepsilon(k_D) = \hbar\omega(k_D) \Pi_n(k_D) \quad (3)$$

Steady state then requires  $\Pi_\varepsilon = \hbar\omega(k_D)\Pi_n$  at all  $k$  in the inertial range; for our  $\omega(k)$ , this means that  $\Pi_\varepsilon \propto k_D^2\Pi_n$ . To formally derive Eq. 3, one multiplies Eq. 1 by  $\hbar\omega(k)$  and invokes the continuity equation for the energy to obtain

$$\frac{\partial \Pi_\varepsilon(k, t)}{\partial k} = \hbar\omega(k) \frac{\partial \Pi_n(k, t)}{\partial k} \quad (4)$$

in the inertial range. For  $k < k_D$ , this equation is trivially satisfied by both of its sides being zero, and does not impose any relation between  $\Pi_\varepsilon(k)$  and  $\Pi_n(k)$ . However, integrating it across a shell around  $k_D$ , and setting  $n(k)$  and all fluxes to zero for  $k > k_D$ , recovers Eq. 3.

Experimentally, we vary  $k_D$  while keeping  $F_0$  fixed and measure  $\Pi_n(k_D)$  according to Eq. 2. To mitigate the effects of the long-term few-percent drifts in the initial  $N$ , and of the additional atom loss through collisions with the background-gas particles, we perform differential measurements of the cascade-induced atom loss,  $N_{\text{loss}}$ , with reference measurements taken by setting  $F_0$  to zero in an otherwise identical experimental sequence.

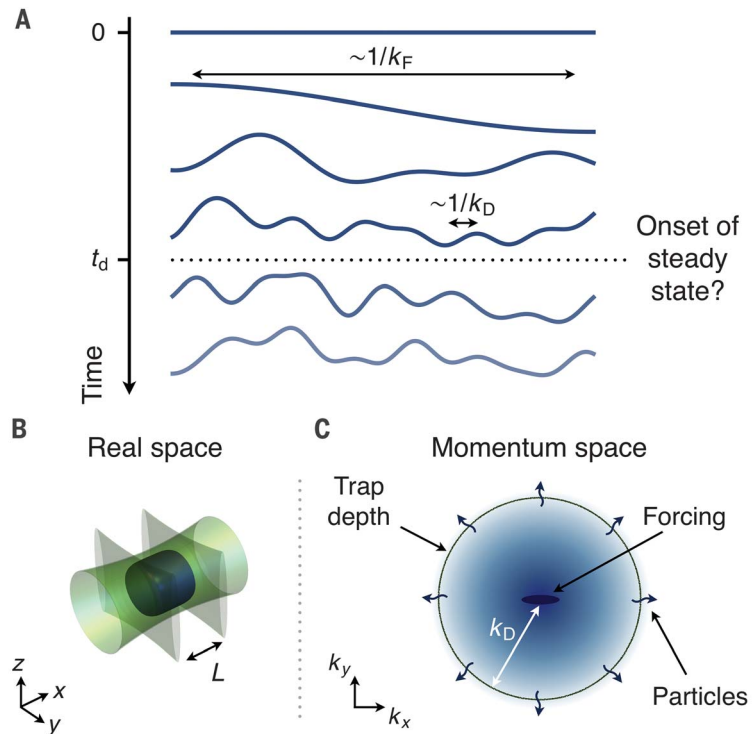
In Fig. 2, we show  $N_{\text{loss}}$  as a function of the shaking time  $t_s$  for various values of  $U_D$ . In all cases at short times, we observe no loss (within error). This is consistent with the expectations that no losses occur at  $k < k_D \propto \sqrt{U_D}$  and that initially it takes time for the excitations to cascade to  $k_D$ , when a steady state can be established (see Fig. 1). For  $t_s$  longer than some onset time  $t_d$ , the loss rate  $\partial N_{\text{loss}}/\partial t$  is essentially constant in time, as long as the total loss is relatively small (<30% of the initial  $N \approx 1.2 \times 10^5$ ). The dashed lines show piecewise linear fits that we use to extract, for each  $U_D$ , both  $t_d$  and the subsequent initial loss rate, which we identify with the steady-state particle flux

$\Pi_n = \Pi_n(k_D)$ . At much longer times,  $t_s \gg t_d$ , the steady-state assumption can no longer hold, because the losses gradually deplete the low- $k$  source of atoms.

In Fig. 3, we show a log-log plot of  $\Pi_n$  versus  $U_D$  (31). We observe power-law behavior with  $\Pi_n \propto U_D^{-1.05 \pm 0.08} \propto k_D^{-2.10 \pm 0.16}$ . We complement these measurements with numerical simulations based on the Gross-Pitaevskii equation, for the same forcing protocol and without any free parameters [see (27) for details]. The

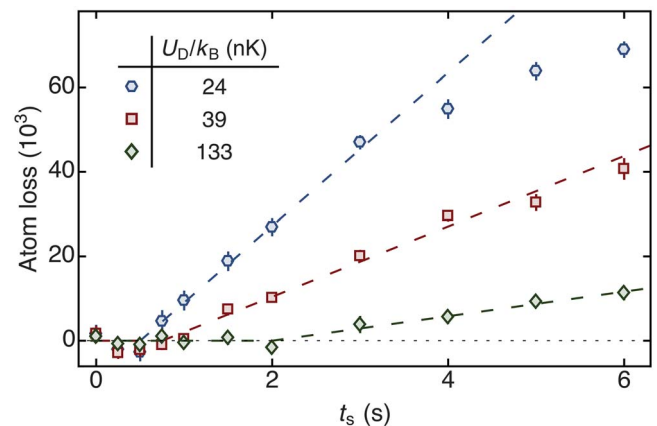
numerical simulation results are shown by solid circles; a fit to the numerical data gives  $\Pi_n \propto U_D^{-1.04 \pm 0.01}$ , in good agreement with the experimental data.

The so-called zeroth law of turbulence, first formulated in the context of classical incompressible fluids, stipulates that for fixed forcing, the steady-state rate of energy dissipation tends to a nonzero constant as the viscosity vanishes ( $\nu \rightarrow 0$ ) (5, 33). In our case, this corresponds to keeping  $F_0$  fixed and taking  $k_D \rightarrow \infty$  (34). This



**Fig. 1. Turbulent cascade in a box-trapped quantum gas.** (A) Cartoon of real-space dynamics of a turbulent wave. Energy is injected by applying a force at a large length scale,  $1/k_F$ , and propagates to smaller scales because of nonlinear interactions. A steady state can be established when the excitations first reach the small dissipation length scale,  $1/k_D$ , at time  $t_d$ . (B) Sketch of the experimental setting. The atoms are trapped in a finite-depth potential in the shape of a cylindrical box, formed by laser barriers. The shaking force is applied along the  $x$  axis. (C) In momentum space, the dissipation scale  $k_D$  is set by the trap depth; when excitations propagate to  $k_D$ , dissipation occurs in the form of particle loss.

**Fig. 2. Atom-loss dynamics associated with the turbulent cascade.** The graph shows atoms lost versus shaking time  $t_s$  for different trap depths  $U_D$  (at  $t_s = 0$ , the atom number is  $N \approx 1.2 \times 10^5$ ). Data points show averages of typically 50 measurements. Dashed lines are piecewise linear fits. The systematic uncertainty in  $U_D$  values is 20%.

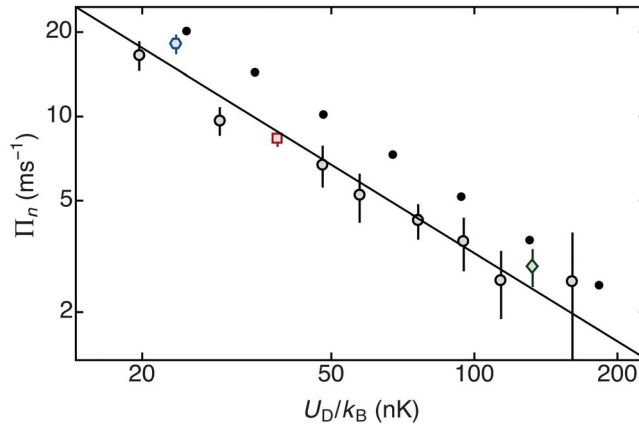


law implies that the particle flux should vanish as  $\Pi_n \sim k_D^{-2}$  (see Eq. 3), in excellent agreement with our data. Note that the steady-state energy balance also requires that  $\Pi_\varepsilon$  is equal to the rate of energy input into the system,  $\varepsilon$ . However, energy conservation alone is not sufficient to predict the scaling of  $\Pi_n$  with  $k_D$ ,

because it is not a priori obvious that for fixed  $F_0$  the rate at which the system absorbs energy from the drive is not affected by changing  $k_D$  (35). Only a posteriori, from Fig. 3 (and the conservation of energy), can we see that in our system the steady-state  $\varepsilon$  must be independent of the dissipation length scale down to

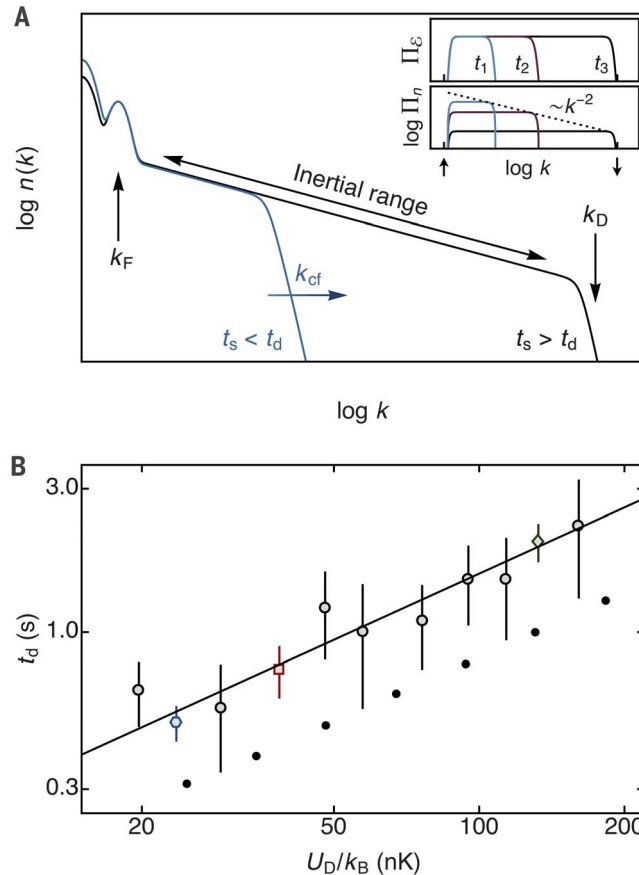
our lowest  $k_D$ . If  $k_D$  were changed dynamically, for a system to reach a new steady state the particle flux would have to self-consistently adjust at all  $k_F < k < k_D$ , because the steady-state  $\Pi_n$  must be both  $k_D$ -dependent (to satisfy the zeroth law) and  $k$ -independent for a given  $k_D$ .

**Fig. 3. Steady-state particle flux.** The atom-loss rate  $\Pi_n$  versus the dissipation energy scale  $U_D$  (open symbols) is shown on a log-log plot; the three colored points correspond to the data shown in Fig. 2. Solid symbols show the results of numerical simulations (27). The systematic uncertainty in  $U_D$  values is 20%. A power-law fit to the experimental data (solid line) gives  $\Pi_n \propto U_D^{-1.05 \pm 0.08} \propto k_D^{-2.10 \pm 0.16}$ , in agreement with the theoretical prediction.



**Fig. 4. Establishing the steady state: The cascade-front dynamics.**

**(A)** Momentum-space turbulent dynamics. Forcing occurs at  $k_F$  and the steady-state distribution  $n(k)$  is established in the wake of the cascade front  $k_{cf}(t_s)$ , which propagates outward until it reaches  $k_D$  at time  $t_d$ . For clarity, here we show an idealized sketch with a very large separation between  $k_F$  and  $k_D$ ; see (27) for numerical simulations with our experimental parameters. Inset: Consistent picture for the evolution of the energy flux  $\Pi_\varepsilon$  and particle flux  $\Pi_n$  for three different times,  $t_1$  (blue)  $<$   $t_2$  (purple)  $<$   $t_3$  (black), with  $t_2 <$   $t_d <$   $t_3$ . The forcing and dissipation scales are indicated by the vertical arrows, as in the main panel. **(B)** Onset time for dissipation. Open symbols show the measured  $t_d$  values versus  $U_D$  on a log-log plot; the three colored points correspond to the data shown in Fig. 2. Solid symbols show the results of numerical simulations (27). The systematic uncertainty in  $U_D$  values is 20%. A power-law fit,  $t_d \propto U_D^\beta$ , to the experimental data (solid line) gives  $\beta = 0.73 \pm 0.06$ , in agreement with the prediction  $\beta = 0.75 \pm 0.05$ .



Having established a consistent picture of the steady-state fluxes in our gas, we now turn to the pre-steady-state turbulent dynamics. In Fig. 4A, we depict the early-time dynamics in Fourier space. The forcing, which generates a surplus of atoms at  $k_F$ , initiates the cascade at  $t_s = 0$ . As the cascade front,  $k_{cf}(t_s)$ , propagates to higher  $k$ , the steady-state momentum distribution,  $n(k) \propto k^{-\gamma}$ , is established in its wake [see also (27)]. The dynamics are dissipationless until  $k_{cf}$  reaches  $k_D$  (at time  $t_d$ ); only then is a steady state, with matching  $\varepsilon$  and  $\Pi_\varepsilon(k_D)$ , established. Hence, our experimental observations of the initial dissipationless stage of turbulence ( $t_s <$   $t_d$ ), and the dependence of  $t_d$  on  $U_D$ , give us access to the dynamics of the cascade front in momentum space.

At  $t_s <$   $t_d$ , the instantaneous particle flux is  $k$ -independent for  $k <$   $k_{cf}(t_s)$ , vanishes for  $k >$   $k_{cf}(t_s)$ , and must match the rate of the population increase in the inertial range:  $n(k_{cf})4\pi k_{cf}^2 dk_{cf} = \Pi_n(k_{cf})dt_s$ , so  $k_{cf}^{2-\gamma} dk_{cf} \propto \Pi_n(k_{cf})dt_s$ . Analogously, for the increase of total energy in the inertial range,  $k_{cf}^{4-\gamma} dk_{cf} \propto \Pi_\varepsilon(k_{cf})dt_s$ , and  $\Pi_\varepsilon(k_{cf})$  is equal to the instantaneous energy injection rate  $\varepsilon$ .

Assuming that  $\varepsilon$ , which we found not to depend on  $k_D$  in steady state, is also independent of  $k_{cf}$  at  $t_s <$   $t_d$ , then the instantaneous  $\Pi_n(t_s)$ , at  $k <$   $k_{cf}(t_s)$ , is  $\propto k_{cf}^{-2}$ . This gives an elegant unifying picture of the particle fluxes for  $t_s <$   $t_d$  and  $t_s >$   $t_d$  [Fig. 4A, inset, and (27)]:  $\Pi_n$  is always the same function of the highest  $k$  for which the steady-state  $n(k)$  has been established (i.e., the lowest  $k$  from which there is no backflow), whether that is the instantaneous  $k_{cf} <$   $k_D$  (for  $t_s <$   $t_d$ ) or  $k_D$ . This self-consistent picture also leads to a quantitative prediction that is verifiable in our experiments: The time independence of  $\varepsilon$  implies  $k_{cf}^{4-\gamma} dk_{cf} \propto dt_s$ , which for  $\gamma <$  5 and  $k_D \gg k_F$  gives a power-law prediction  $t_d \propto U_D^\beta$ , with  $\beta = (5 - \gamma)/2$ . Specifically, for our  $\gamma = 3.5 \pm 0.1$  (9), we predict  $\beta = 0.75 \pm 0.05$ .

In Fig. 4B, we show the variation of  $t_d$  with  $U_D$ . We find that our data are indeed well described by a power law, with  $\beta = 0.73 \pm 0.06$ , in agreement with our prediction. The results of our numerical simulations (solid circles) show similar behavior with a small systematic offset; a fit to the numerical data gives  $\beta = 0.71 \pm 0.01$ .

Finally, we note that the criterion for  $t_d$  to show scaling behavior, namely  $\gamma <$  5 and hence  $\beta >$  0, is intimately linked to another important concept in the theory of turbulence. For  $\gamma <$  5, the steady-state spectrum has infinite



energy capacity, meaning that it carries infinite energy for  $k_D \rightarrow \infty$ . It is indeed generally expected for infinite-capacity systems that the cascade front propagates at a finite speed and that the Kolmogorov-Zakharov turbulence spectrum forms behind it (36).

Our work provides a complete, consistent picture of the wave turbulence dynamics at both short (pre-steady-state) and long (steady-state) times. In the broader context of far-from-equilibrium many-body quantum systems, a turbulent quantum gas with a large  $k_D$  provides a particularly interesting example of an essentially stationary nonthermal state [see also (37–39)]. The possibility of synthetic dissipation also opens new theoretical perspectives. In the future it would be interesting to engineer arbitrary momentum-cutoff landscapes, which could, for example, allow studies of anisotropic turbulence. By dynamically tuning the dissipation scale or the driving force, it should also be possible to study quenches between different turbulent states.

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32. Note that it is the total radial flux  $\Pi_r$ , rather than  $|\Pi_r|$ , that is  $k$ -independent in the inertial range.
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#### SUPPLEMENTARY MATERIALS

science.sciencemag.org/content/366/6463/382/suppl/DC1  
Supplementary Text  
Figs. S1 to S3  
References (41–44)

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## Synthetic dissipation and cascade fluxes in a turbulent quantum gas

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### Down and down the energy cascade

Injecting energy into a turbulent system at large length scales results in the energy cascading down and eventually dissipating at a characteristic small length scale. In conventional fluids, this small scale is set by fluid viscosity. Navon *et al.* studied the turbulence energy cascade in a quantum gas, a Bose-Einstein condensate of rubidium-87 atoms held in a uniform trap. Dissipation occurred by atoms escaping from the trap at a scale that could be tuned by varying the height of the trapping potential. Thanks to the flexibility of their setup, the researchers were able to study both the steady state, in which energy is injected at the same rate it is dissipated, and the transient regime preceding the steady state.

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