Many-Body Decay of the Gapped Lowest Excitation of a Bose-Einstein Condensate

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We study the decay mechanism of the gapped lowest-lying axial excitation of a quasipure atomic Bose-Einstein condensate confined in a cylindrical box trap. Owing to the absence of accessible lower-energy modes, or direct coupling to an external bath, this excitation is protected against one-body (linear) decay, and the damping mechanism is exclusively nonlinear. We develop a universal theoretical model that explains this fundamentally nonlinear damping as a process whereby two quanta of the gapped lowest excitation mode couple to a higher-energy mode, which subsequently decays into a continuum. We find quantitative agreement between our experiments and the predictions of this model. Finally, by strongly driving the system below its (lowest) resonant frequency, we observe third-harmonic generation, a hallmark of nonlinear behavior.

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Much of our understanding of interacting quantum many-body systems is rooted in the existence of long-lived elementary excitations. The nature of these excitations reflects the form of order in the underlying state of matter. Moreover, the excitation energy spectrum gives access to the low-temperature thermodynamics [1,2]. To lowest order, it is typically calculated by assuming that the excitations are noninteracting and, thus, have an infinite lifetime.

In the continuum limit, taking into account the interactions between quasiparticles generically leads to finite lifetimes, even at zero temperature [3]. Such lifetimes play a crucial role in understanding transport properties of quantum fluids, such as their thermal and electrical conductivity, viscosity, or the attenuation of sound. The case of gapped excitations is profoundly different, owing to additional constraints on decay channels. Gaps in the energy spectra naturally arise in systems with discrete translational symmetry [4], or as a result of many-body [5–7] and finite-size effects [8,9]. The problem of the lifetime of gapped excitations has been considered in myriad contexts, including optical phonons in crystals [10], quasiparticles in quantum dots [11], magnons in spin systems [12,13], and rotons in superfluid helium [14–16]. However, the decay considered in these studies typically originated from the coupling to lower-lying modes (e.g., acoustic branches), or scattering due to thermally populated excitations.

In this Letter, we study the fundamental many-body problem of the lifetime of a gapped lowest-lying excitation at zero temperature. This situation is particularly intriguing since the excitation is energetically immune to any one-body (i.e., linear) decay. The finite lifetime exclusively arises from nonlinear effects and provides a window into the interactions between quasiparticles. We experimentally and theoretically study this problem in the textbook setting of a homogeneous box-trapped atomic Bose-Einstein condensate (BEC).

The weakly interacting bulk Bose gas has been a remarkable test bed for exploring the physics of excitations and their interactions [17–44]. At low temperature, the excitations of an infinite uniform BEC have an energy given by the Bogoliubov spectrum; in the long-wavelength limit, these excitations are phonons [47]. The leading decay channel for the phonons is a linear process, in which they spontaneously break up into pairs of longer-wavelength phonons [Fig. 1(a)], a damping mechanism first predicted by Beliaev [48].

By contrast, in our case, the finite system size leads to experimentally resolvable gaps in the excitation spectrum, and recent works suggested that the damping of the lowest mode is fully nonlinear (within experimental precision) [41,49]. Here, we experimentally and theoretically elucidate the excitation lifetime of this mode by investigating the...
steady-state response of the system to a continuous drive. We show that the underlying damping mechanism can be explained by a generic microscopic model based on an inverse Beliaev-like process, whereby two elementary excitations merge into a higher-energy one [Fig. 1(b)].

Our experiment starts with a quasiuniform BEC of $^{87}$Rb atoms [50], prepared in a cylindrical optical box trap of radius $R = 16(1)$ μm and length $L = 26(1)$ μm [see Fig. 2(a)]. Our condensates consist of $N = 1.2(1) \times 10^5$ atoms, their chemical potential is $\mu \approx k_B \times 2.0(3)$ nK, their healing length $\xi \approx 1.2(1)$ μm, and the excitation frequency of the lowest-lying axial mode [Fig. 2(b)] is $\omega_{d} = 2\pi \times 9.0(1)$ Hz (see [41] for details). We excite this mode by applying a spatially uniform oscillating force $F(t) = (U_s/L) \sin(\omega t)e_z$, where $e_z$ is the unit vector along the symmetry axis of the cylinder and $U_s$ is the maximal potential-energy drop across the box. The force is produced by a pair of coils, which create a magnetic field gradient that couples to the magnetic moment of the atoms. Following a variable shaking time $t_s$, the BEC is released and the center of mass of the atomic density distribution is recorded after a time of flight $t_{\text{OF}} = 140$ ms, which reflects the in situ center-of-mass velocity $v(t_s)$ along $e_z$.

In Fig. 2(c), we show examples of $v(t_s)$ for three drive frequencies (below, on, and above resonance), and for two drive amplitudes ($U_s/k_B = 0.2$ and 1.6 nK). While transient dynamics are visible at early times, a monochromatic steady state is established at later times. We fit the data for $t_s > 0.6$ s with $v(t_s) = v_m \sin(\omega t_s + \theta)$ (solid lines) and extract the amplitude $v_m$ and phase $\theta$ of the velocity response. For the weaker drive ($U_s/\mu \approx 0.1$), we observe significant response only on resonance. However, for the stronger drive ($U_s/\mu \approx 0.8$), we observe comparable response amplitudes at all three frequencies, indicating significant broadening of the response spectrum, a clear signature of nonlinear behavior. Specifically, such broadening corresponds to the shortening of the lifetime of excitations due to their mutual interactions.

To characterize this nonlinear behavior, we plot $v_m$ as a function of $U_s$ on log-log scale for various drive frequencies in Fig. 2(d). For linear response, with amplitude-independent damping, one would get $v_m \propto U_s$, for all $\omega$. Instead, on resonance ($\omega = \omega_d$), we observe power-law behavior $v_m \propto U_s^{1/3}$, reminiscent of classical models with a cubic nonlinear damping (which generically predict $v_m \propto U_s^{1/3}$ on resonance; see Sec. I in [51]). Away from resonance, $v_m$ is smaller than on resonance (for the same $U_s$), but for stronger drives, a progressively broader range of frequencies becomes effectively resonant.

In the following, we introduce a theoretical model for this nonlinear behavior, and compare it to the full experimental response curves $v_m(\omega)$ and $\theta(\omega)$ (see Fig. 3).

Within quantum theory, in the spirit of Fig. 1(b), we associate the cube-root scaling $v_m \propto U_s^{1/3}$ with the decay of the mode toward higher energies via a two-body process [52], resulting in a decay rate proportional to the number of
operators for the fundamental and the auxiliary mode, $\lambda$ is the coupling strength, and $\Omega$ is the strength of the drive. We incorporate the decay of the auxiliary mode into the quasicontinuum via a master equation approach. By tracing out the auxiliary mode, we derive, within a mean-field approximation, an equation of motion for the mean dipole $d(t) = \langle \hat{d}(t) \rangle$:

$$i\partial_t d - (\omega_d + \kappa_2 |d|^2) d = \Omega \sin(\omega t).$$

The generally complex $\kappa_2$ captures the nonlinear effects to leading order. Specifically, $\text{Re}[\kappa_2]$ and $\text{Im}[\kappa_2]$ correspond, respectively, to a frequency shift (due to the self-interaction) and a nonlinear damping (due to the mediated coupling to the continuum); expressions for $\kappa_2$ in terms of the microscopic model parameters are provided in Sec. III in [51]. In practice, $\kappa_2$ is sensitive to the details of the trapping potential, and it is more convenient to extract it directly from the experimental data. However, the form of Eq. (2) is universal in that it does not depend on the exact loss mechanism of the auxiliary mode, nor the number of auxiliary excitations involved in the elementary interaction process [see Eq. (1)].

To compare our experimental data to the theory, we relate $d(t)$ to the main experimental observable $\nu(t) = (2\alpha/L) \partial_t \text{Re}[\langle \hat{d}(t) \rangle]$, where $\alpha$ is the matrix element of the position operator $\hat{z}$ between the ground state and the lowest-lying excitation (see Sec. IV in [51]). In terms of the experimental parameters, the drive amplitude in Eq. (1) is $\Omega = \alpha U_s/\hbar L$.

We determine the parameters of the model by fitting the $\nu_m(\omega)$ response curves to the steady-state numerical solutions of Eq. (2) for each $U_s$. The resulting fits are shown in Fig. 3(a) as solid lines, where, for simplicity, we first neglect the real part of the nonlinear coefficient $\kappa_2$, so that the adjustable parameters are $\bar{\kappa}_2 = i\kappa_2$ and $\alpha$. We see that for $U_s \lesssim k_B \times 2$ nK, the fitted model captures the experimental data well. Only for $U_s \gtrsim k_B \times 2$ nK do deviations between the model and the data become apparent. In the inset of Fig. 3(a), we plot the extracted full width at half maximum of the spectral lines, $\Gamma$, which is inversely proportional to the excitation lifetime, as a function of $U_s$. The plot reveals that the deviation between the model and the data occurs only once $\Gamma \gtrsim \omega_d$. For $U_s/k_B = 1.6$ nK, we estimate that $\bar{\kappa}_2 |d|^2 \approx 0.5 \omega_d$, and higher-order nonlinearities could become important.

The parameters extracted from each $\nu_m(\omega)$ curve are shown in Figs. 3(b) and 3(c). Crucially, both $\alpha$ and $\bar{\kappa}_2$ do not depend on $U_s$ within experimental errors, demonstrating that the model [Eq. (2)] captures the nonlinear $U_s$-dependent response. Averaging the fitted parameters within the range of validity of the model ($U_s/k_B < 1$ nK), we obtain $\alpha/(L/\sqrt{N}) = 0.098(4)$ and $\bar{\kappa}_2/\omega_d = 6.9(7) \times 10^{-6}$ per phonon (solid horizontal lines).

A calculation assuming a cylindrical-box-trapped BEC in the Thomas-Fermi regime (Sec. III in [51]) yields...
of phase with the drive below (above) resonance. [Fig. 3(b)], which is close to the experimental value. \( v_m = U_s \) (harmonic oscillator), one still recovers \( v_m = \frac{\omega}{(2\pi)} = 9 \) Hz [53]). However, the behavior is markedly different for \( \omega/(2\pi) = 3.5 \) Hz (bottom panels), for which \( 3\omega \) is close to the resonant \( \omega_d \). In this case, the Fourier spectrum of the time-domain response [Fig. 4(b)] shows a clear peak at \( 3\omega \), signaling third-harmonic generation, in addition to the main peak at \( \omega \) [54].

In conclusion, we have experimentally and theoretically studied the nonlinear decay of the fundamental gapped excitation of a Bose-Einstein condensate. Our experiments reveal a cubic damping mechanism as well as third-harmonic generation, which we have shown can be captured by a mean-field model based on a microscopic theory at lowest order. The decay of lowest-lying gapped excitations is an understudied but generic problem in quantum many-body physics, and in the future, it would be interesting to transpose this study to other many-body systems such as the strongly interacting Fermi gas [56,57], dipolar gases [58–60], or topological systems [61]. We note that for our strongest (resonant) drive the gas is turbulent in steady state [49], and the many-body decay of the lowest-lying mode provides a stepping stone toward the transfer of energy to higher-lying modes. Thus, our work paves the way for a microscopic understanding of the genesis of a turbulent cascade [49,62], where the energy leakage from the driven discrete lowest-lying mode is sufficiently large to sustain a nonequilibrium steady state. Such a transition from the discrete-state dynamics to a continuum turbulent cascade has recently been theoretically studied in a cosmological context [63]. Finally, the exclusively nonlinear decay of the lowest-lying mode opens up the prospect of exploring quantum state preparation and the generation of nonclassical states of this degree of freedom, analogous to the quantum control of a single mode in optomechanics [64–66].

Data supporting this publication are available in the Apollo repository [67].

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[44] For related studies on excitations of ultracold gases in optical lattices, see, e.g., [45,46].


[52] More generally, a $M$-body process of this form would lead to a $1/(2M-1)$ power-law scaling (see Sec. VII in [51]).

[53] In this case, the response amplitude still follows the resonant scaling $v_\omega \propto U_1^{1/3}$ seen in Fig. 2(b).

[54] Note that the resonance condition for third-harmonic generation, $3\omega = \omega_d$, is not stringent because of the strong nonlinear broadening; indeed, for our strongest drive, we observe a weak third-harmonic generation signal even for $\omega/(2\pi) = 5$ Hz.

[55] We extract $F_3$ by fitting the data with $v(t_s) = F_1 \sin(\omega t_s + \theta_1) + F_3 \sin(3\omega t_s + \theta_3)$.


[67] https://doi.org/10.17863/CAM.63897.