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Non-Hermitian topological phases in an extended Kitaev model

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Abstract. In this work we address the study of topological phase protection of open quantum systems. Using the self-energy formalism, we investigate the paradigmatic case of an extended Kitaev model. The results show how the topological order can be affected by coupling the system to two external leads, giving rise to Non-Hermitian topological phases. Our results could be useful in spectroscopic measurements made on nanowire-based mesoscopic devices.

1. Introduction

One of the recent efforts, in condensed matter physics, is to implement fault-tolerant quantum gates using Majorana zero-modes (MZMs), quasiparticles of topological superconductors [1]. The detection of non-Abelian statistics of MZMs, the measurement of conductance that is strictly connected to parity of the systems [3] and the implementation of braiding dynamics in superconducting nanowires, i.e dynamical processes that exchange Majorana fermions [4, 5] are some of the salient topics developed in the last decade.

Albeit closed systems hosting MZMs have been extensively studied and discussed in the literature (see e.g. [6, 7, 8, 9]), it remains relevant to understand how the topological phases are modified by measurement procedure which can be realized by coupling the system to a source and a drain. To this aim, using the analysis of the existence of exceptional points in the self-energy, we study the topological phases of the paradigmatic case of a Kitaev ladder [7] by coupling this system to a normal electrode and a p-wave superconducting electrode. We show how the topological order of closed system can be changed as an effect of the opening of the system, giving rise to Non-Hermitian topological phases.

The paper is organized as follows. In Sec. 2 we summarize the theory of exceptional points and non-Hermitian topology and we apply the general features to our model. The numerical simulations are shown in Sec. 3. Finally, the conclusions are given in Sec. 4.

2. Exceptional points and Non-Hermitian topology

It is known in closed 1D topological superconducting systems that the presence of protected Majorana modes is expressed by a non trivial topological invariant Q. By means of the bulk-edge correspondence, the topological order of closed systems is discussed by looking at the band

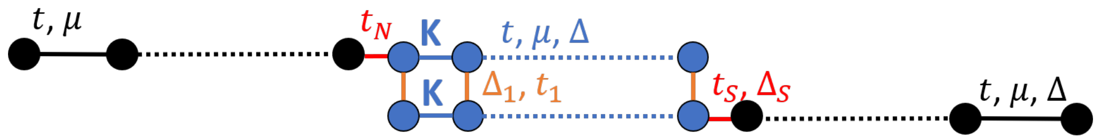

Figure 1. Schematic representation of the system's setup. The sites are represented by circles, the tight binding parameters by links. Each colour corresponds to a component of the system. Blue refers to the single Kitaev chain, black refers to the leads, red to coupling parameters between the leads and the ladder, orange refers to the interchains coupling.

topology. For open quantum systems the same information can be obtained in the frame of non-Hermitian topology defined in terms of the complex poles ϵ of the retarded Green's function:

$$G^r(\omega) = [\omega - H_{eff}(\omega)]^{-1} \tag{1}$$

where $H_{eff}(\omega)$ is an effective non-Hermitian Hamiltonian $H_{eff}(\omega) = H_0 + \Sigma(\omega)$ which describes the bare system H_0 and the coupling with the electrodes $\Sigma(\omega)$ (self-energy).
The poles of the retarded Green's function are the complex eigenvalues of H_{eff}: $\epsilon = E - i\Gamma$ where E represents the real energy of H_0 and Γ the coupling with the leads. By virtue of particle-hole symmetry, which is satisfied by the superconducting systems, the complex eigenvalues must come in pairs: $\epsilon, -\epsilon^*$ [10]. This statement means that the eigenvalues can be located symmetrically at opposite sides of the imaginary axis or, alternatively, lay exactly on the imaginary axis. The latter correspond to non trivial modes (MZMs) while the former are trivial modes with finite-energy. A bifurcation of two trivial modes in two non trivial one, when we vary the parameters that induce the topological phase transition, defines an exceptional point. By means of this formalism, we can study the topological phase transitions by looking at the bifurcation points of the imaginary parts of the complex eigenvalues.
We consider our system, the Kitaev ladder, coupled to a normal lead end a superconducting p-wave lead to form an in-line configuration, i.e. with the first site of the fist chain of the system coupled via an hopping strength t_N to the normal lead and the last site of the second chain coupled to the p-wave lead via an hopping and pairing strength t_S, Δ_S. The illustration of the model is shown in Fig. (1). We indicate with H_N, H_L, H_S respectively the Hamiltonians of the normal lead, the Kitaev Ladder and the p-wave lead:

$$\begin{cases} H_N = \sum_{j=1}^{L}[(-ta_j^\dagger a_{j+1} + h.c.) - \mu_N c_j^\dagger c_j] \\ H_L = H_{K_1} + H_{K_2} + H_{K_1,K_2} \\ H_S = \sum_{j=1}^{L}[(-tc_j^\dagger c_{j+1} + \Delta c_j c_{j+1} + h.c.) - \mu_S c_j^\dagger c_j] \end{cases} \tag{2}$$

where H_{K_1} and H_{K_2} are the Hamiltonians of two isolated Kitaev chains [11]:

$$H_{K_\alpha} = \sum_{j=1}^{L}[(-tb_{j,\alpha}^\dagger b_{j+1,\alpha} + \Delta b_{j,\alpha} b_{j+1,\alpha} + h.c.) - \mu b_{j,\alpha}^\dagger b_{j,\alpha}]$$

where $\alpha = 1, 2$ is the chain index and H_{K_1,K_2} describes the interchain coupling whose expression is given by:

$$H_{K_1,K_2} = \sum_{j=1}^{L}\left[-t_1 b_{j,1}^\dagger b_{j,2} + \Delta_1 b_{j,1} b_{j,2} + h.c.\right]. \tag{3}$$

a_j, b_j, c_j ($a_j^\dagger$, $b_j^\dagger$, $c_j^\dagger$) are the fermionic annihilation (creation) operators respectively for normal lead, system and p-wave lead. The labels $1, 2$ in Eq. (3) denote the two chains of the ladder, j is

the site index, $t > 0$ is the amplitude of the nearest-neighbour hopping, $\Delta > 0$ is the amplitude of the superconducting pairing, μ represent the chemical potential of the ladder chains while μ_N, μ_S the chemical potential of the two leads and t_1, Δ_1 are the transversal hopping and pairing amplitude. For simplicity, we are choosing equal tight binding parameters t, Δ for system and leads.
Introducing the Nambu spinor formalism: $\Psi_{N/S} = (a_1/c_1, a_1^\dagger/c_1^\dagger, ..., a_N/c_N, a_N^\dagger/c_N^\dagger)^T$, $\Psi_L = (b_{1,1}, b_{1,1}^\dagger, ..., b_{N,2}, b_{N,2}^\dagger)^T$ we can write Eq.s (2) into the BdG form [12]:

$$\begin{cases} H_N = \frac{1}{2}\Psi_N^\dagger H_1 \Psi_N \\ H_L = \frac{1}{2}\Psi_L^\dagger H_2 \Psi_L \\ H_S = \frac{1}{2}\Psi_S^\dagger H_3 \Psi_S \end{cases} \tag{4}$$

where H_1, H_2 and H_3 are $4L \times 4L$ BdG Hamiltonians, where L is the number of sites of every chain. The full wavefunction $\psi = (\psi_1, \psi_2, \psi_3)^T$, describing the system and the leads, satisfies the following equation:

$$i\hbar\partial_t \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \begin{pmatrix} H_1 & V_{12} & 0 \\ V_{21} & H_2 & V_{23} \\ 0 & V_{32} & H_3 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \tag{5}$$

where V_{12}, V_{21} describe the coupling between Hamiltonians H_1 and H_2, V_{23} and V_{32} between H_2, H_3: $V_{12} = -t_N a_L^\dagger b_{1,1} + h.c.$, $V_{23} = -t_S b_{L,2}^\dagger c_1 + h.c.$ and the hermitianity condition implies $V_{12}^\dagger = V_{21}$ and $V_{23}^\dagger = V_{32}$. By means of some straightforward algebra steps on the system of equations (5) and resorting to the Fourier transform of the vectors: $\psi_i = \int \frac{dE}{2\pi\hbar}\psi_i(E)e^{-i\frac{Et}{\hbar}}$, we obtain the equation for the wave function of the system (ψ_2) whose Hamiltonian, however, takes into account the coupling with the leads:

$$i\hbar\partial_t\psi_2 = H_2\psi_2 + \int dt' \Sigma(t-t')\psi_2(t') \tag{6}$$

where $\Sigma(t-t') = \int \frac{dE}{2\pi\hbar}\Sigma(E)e^{-i\frac{E}{\hbar}(t-t')}$ is the self-energy and $\Sigma(E)$ is given by:

$$\Sigma(E) = V_{21}(E-H_1)^{-1}V_{12} + V_{23}(E-H_3)^{-1}V_{32}. \tag{7}$$

Assuming that $\Sigma(E)$ depends weakly on E near to the Fermi energy E_F: $\Sigma(E) \approx \Sigma(E_F)$, which is the region where the BdG formalism works, we obtain:

$$\Sigma(t-t') = \Sigma(E_F)\delta(t-t'), \quad i\hbar\partial_t\psi_2 = H_{eff}\psi_2$$

where $H_{eff} = H_2 + \Sigma(E_F)$ is the effective Hamiltonian for our model in the self-energy formalism and describes the ladder H_2 and the coupling with the leads $\Sigma(E_F)$:

$$\Sigma(E_F) = V_{21}(E_F-H_1)^{-1}V_{12} + V_{23}(E_F-H_3)^{-1}V_{32} \tag{8}$$

3. Bifurcation of exceptional points: numerical results

Using the self-energy theory presented above, we have studied the bifurcation of the poles (exceptional points) of the ladder model. In Fig. 2 we present the topological phase diagram of the isolated ladder (for more details see [7]) in the (t_1, μ) plane by fixing $\Delta = 0.8$ and setting two different values of $\Delta_1 = 0.09,\ 0.8$, while taking t as the energy unit. The integers in the panels of Fig. 2 correspond to the number of MZMs at every edge of the ladder. We note the

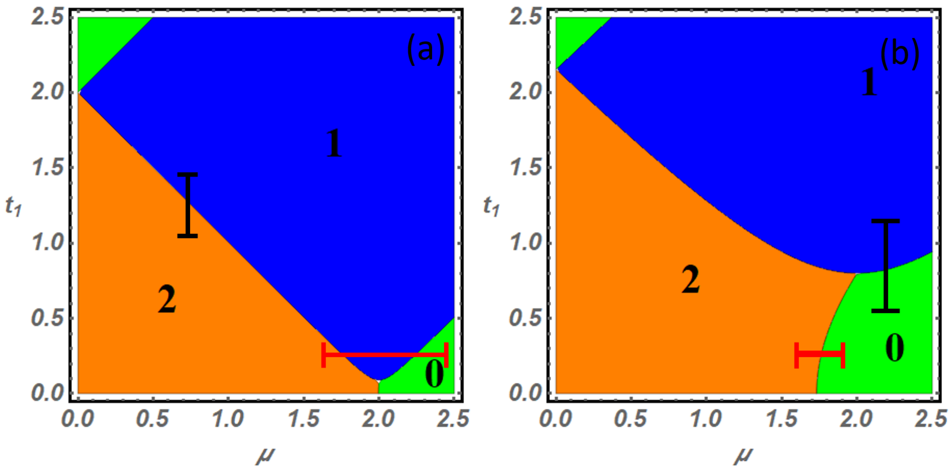

Figure 2. Topological phase diagram of the ladder in the (t_1, μ) plane, given by the winding number for two different values of $\Delta_1 = 0.09,\ 0.8$ (respectively panel (a) and (b)) and for $\Delta = 0.8$, $t = 1$, $L = 150$. The orange, blue and green regions are respectively the regions with 2, 1 and 0 MZMs per edge. The black and red lines represent the cuts on which we evaluate the bifurcation of exceptional points. The panels (a) and (b) are extracted from paper [7].

richness of the topological phase plane with three different phases and the possibility, increasing Δ_1 to produce an enlargement of the trivial region and changing μ and setting appropriate values of t_1 to have a direct transition from 2 MZMs region to the trivial region without crossing the 1 MZMs phase. Resorting to numerical analysis, by means of Eq. (8), we investigate what happens to H_{eff}. In particular, Fig. 3 show the lowest real part in energy and the corresponding imaginary part of the complex eigenvalues pursuing the horizontal red cuts (panels (a), (b)) and the vertical black cuts ((c), (d)) of Fig. 2, where the following topological phase transitions are expected to occur: $2-1-0$, $2-0$, $2-1$, $0-1$. The analysis of the complex eigenvalues evidences that the bifurcation of imaginary part, appears at the transition between the topological phase and the trivial one, but not at the transition between the two topological phases with one and two MZMs, causing the loss of transition between these two phases. The opening of the system to leads, produces a degradation of the initial topological properties and give rise to a "poorer" topological phase diagram. A similar trend was observed in [13] where the results show how the signatures of topological order affect the electrical response of the analized systems.

Panels (a) and (b) of Fig. 4 show the square modulus of the only topological mode of H_{eff}, in two specified points of the panel (a) of Fig. 2 ($t_1 = 0.2$, $\mu = 1.7, 2.1$) where the isolated ladder has respectively two modes and one mode. Panels (c), (d) show the local Majorana polarization [14] $P_M(\omega, n) = 2\sum_m \delta(\omega - \epsilon_m) u_n^{*(m)} v_n^{(m)}$, where u_n (v_n) is the electron (hole) n component of the mth eigenstate of the system in the Nambu representation, which is an order parameter to characterize the Majorana "nature" of a quantum state. In fact, it has been shown [15] that the full Majorana polarization $P_M(\omega) = \sum_{n=1}^{L/2} P_M(\omega, n)$, where L is taken even, is zero for non-Majorana modes and -1 or $+1$ for genuine Majorana states. This modes analysis confirm that despite the 2 MZMs phase and the 1 MZMs phase of the isolated system, only 1MZM is robust against the hybridization process with the electrodes.

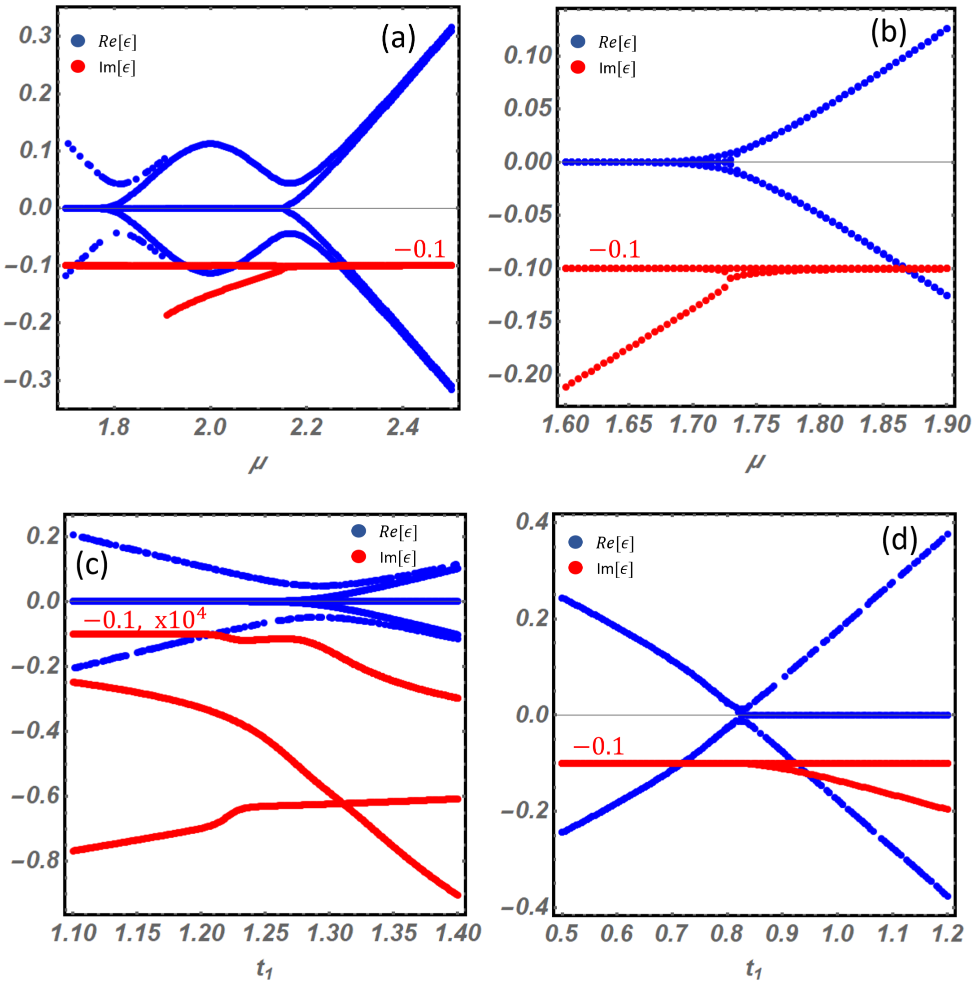

Figure 3. Plots of the real part (blue curves) and imaginary part (red curves) of the lowest eigenvalues in energy of H_{eff}. Panel (a) and (b) are taken following respectively the two horizontal red cuts of Fig (2) while panels (c), (d) following respectively the two vertical black cuts. In panels (a), (b) $t_1 = 0.2$, in panel (c) $\mu = 0.7$ and in panel (d) $\mu = 2.2$.

4. Conclusions

We have addressed the problem of topological phase characterization of an open quantum system by studying the paradigmatic case of a Kitaev ladder in the self-energy formalism. We have found that the coupling with the leads produces a degradation of the topological properties of the isolated system with more than one MZMs per edge and that only the phase transition between the trivial and the topological phase with one MZMs is clearly seen. These findings could be useful in the interpretation of spectroscopic measurements where more than one peak, whose width is related to the imaginary part of the self energy, could appear. Larger the width of such peaks is, more the imaginary part of the self energy dominates.

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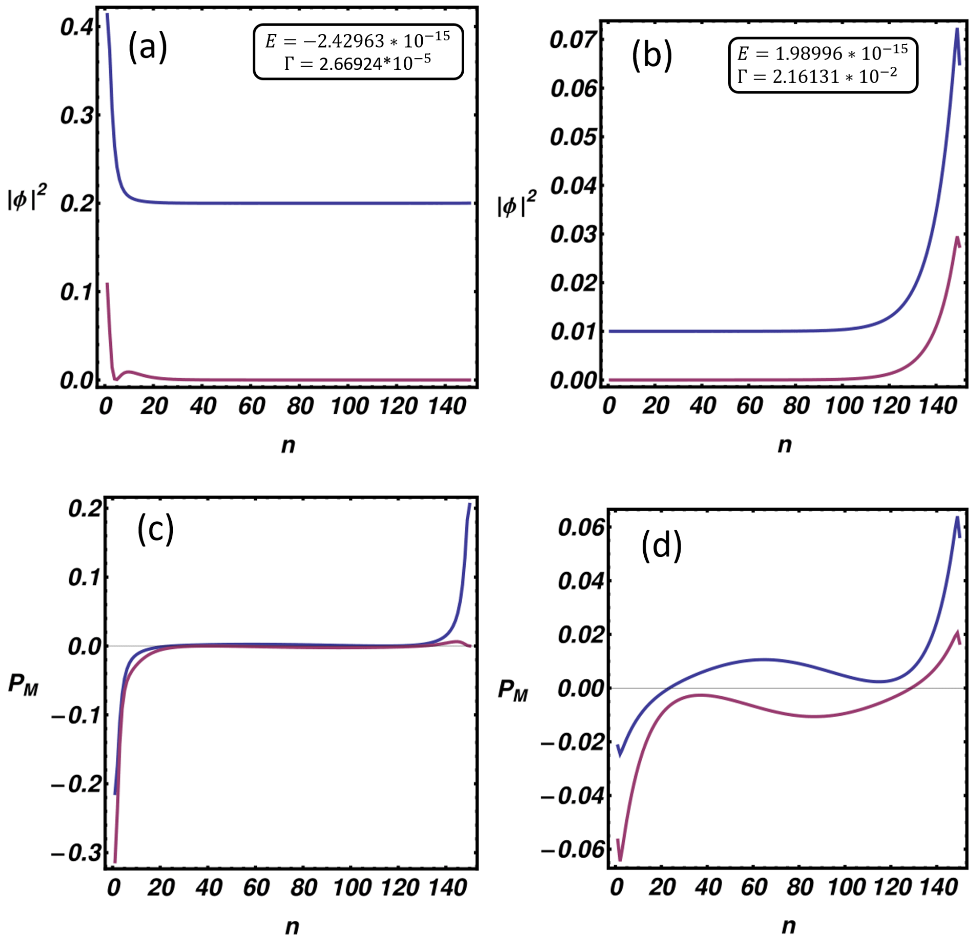

Figure 4. Square modulus of the only topological mode of H_{eff} in the 2 MZMs phase $t_1 = 0.2$, $\mu = 1.7$ (panel (a)) and 1MZMs phase $t_1 = 0.2$, $\mu = 2.1$ (panel (b)) of Fig 2. Panels (c) and (d) show the local Majorana polarization at the same two points of the phase diagram of Fig 2. The blue plots and the purple plots are respectively the modes along the first and the second chain of the ladder.

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