CoQuaDis: Collective quantum phenomena in dissipative systems – towards time-crystal applications in sensing and metrology

Partners

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Atomic physics	Synchronisation	Trapped ion physics
Mathematical physics	Quantum thermodynamics	Quantum information
Many-body physics	Feedback control & ML	Quantum measurement

1 Project idea

Although dissipative processes appear to be detrimental to quantum coherence, their competition with quantum interactions can lead to **collective dynamical behaviour** and coherence times that exceed the bare rate of the dissipative process by orders of magnitude. The advantage of these collective phenomena is that they do not rely on perfect coherence from the outset and are expected to yield a certain degree of robustness against perturbations.

The overarching goal of this research project is to **theoretically analyse**, to **experimentally study**, and to lay out the **pathway to practically exploit** collective effects in dissipative many-body systems.

2 Physical system

We will use a **cold ion platform** to realize spin-boson models which feature collective effects. The trapped ions form a **Wigner crystal** at sufficient low temperatures.

- **Spin** degrees of freedom represent the internal electronic structure of the ion.
- **Bosonic** modes encode the vibrational modes of the ion crystal and are used to engineer interactions between the spins.

This platform allows to generate and investigate a dissipative many-body system, which is highly controllable. Dissipation further provides the possibility to continuously read-out the system state**.**

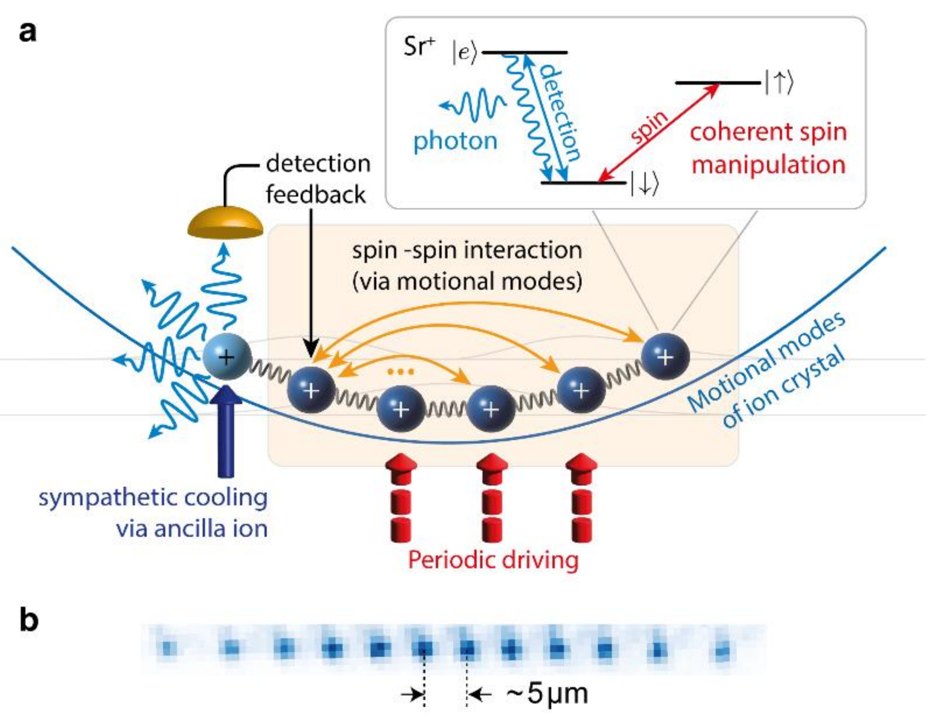

Fig. 1. **Realisation of a spin-boson model in a trapped ion system**. (a) Trapped ions forming a Wigner crystal at low temperatures. (b) Camera image of an experimental realisation of a linear ion string with 12 ions (Stockholm University).

3 Nonequilibrium phases and continuous monitoring

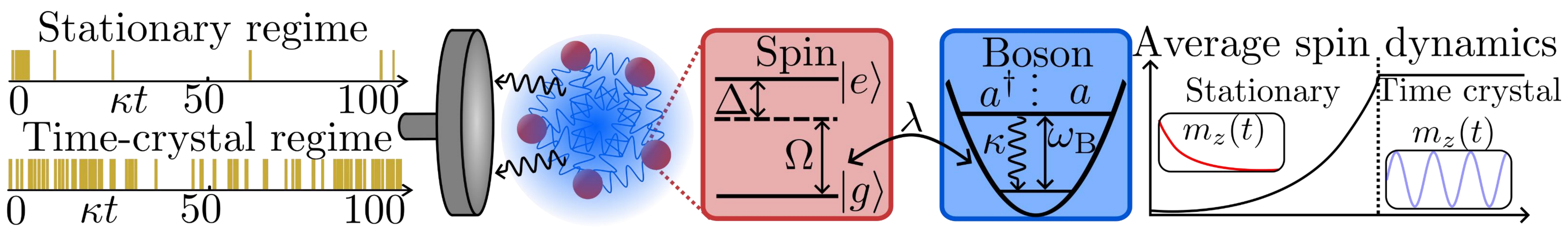

Fig. 2. **Spin-boson model, nonequilibrium phases and measurement output**. An ensemble of spins, driven by a laser with Rabi-frequency Ω, coupled to a (lossy) bosonic mode and featuring a nonequilibrium phase transition from a stationary to a time-crystal phase.

We focus on novel phases of matter that can occur in **out-of-equilibrium scenarios** and cannot be captured by the canonical framework of thermal equilibrium states. In particular, we are interested in **emergent many-body phenomena** that occur in the presence of phase transitions. One prominent example are the so-called **time-crystals (TC)**, which constitute a non-stationary phase which is **stabilised by many-body effects.**

The nonequilibrium setting allows for **continuous monitoring** of the system, where the specific **unravelling** is determined by the measurement performed on the system. The state of the system is **conditioned** on the **measurement record** and can be evolved with the associated **Kraus operators. Average properties** of the system are recovered by averaging over several **realizations/trajectories**. Experimentally relevant unravellings are:

- **Photon counting**: $\mathbf{i_t} = \{i_1, \dots, i_M\} \leftrightarrow K_0 = e^{-i\left(H - \frac{i}{2}L^\dagger L\right)\Delta t}, \quad K_1 = \sqrt{\Delta t} L$
- **Homodyne detection**: $\mathbf{i_t} = \{J_1, \dots, J_M\} \leftrightarrow K_{J_j} = \mathbf{1} - iH\Delta t - \frac{L^\dagger L}{2}\Delta t + Le^{i\Phi} J_j \Delta$

4 Many-body enhanced sensitivity with classical measurements

The precision to which a system parameter $\boldsymbol{\omega}$ can be optimally estimated from the measurement record $\boldsymbol{i_t}$ is fundamentally bounded by the so-called Cramér-Rao bound $(\delta\omega)^2 \geq [N_{\text{reps}}\mathcal{F}_{\text{cl}}(\omega,t)]^{-1}$, where the classical *Fisher information* (FI) $\mathcal{F}_{\text{cl}}$ is derived from the conditional probability $p(\mathbf{i}_t|\omega)$. Ultimate knowledge about $\boldsymbol{\omega}$ is given by considering the combined system-environment state

$$|\Psi(t)\rangle = \sum\nolimits_{\mathbf{i_t}} K_{i_M} \dots K_{i_1} |\psi(0)\rangle \otimes |i_M, \dots, i_1\rangle$$

and optimizing over all possible measurements, which leads to the *Quantum Fisher information* (QFI)

$$\mathcal{F}_{\text{SE}}(\omega,t) = 4(\langle \partial_\eta \Psi(t) | \partial_\eta \Psi(t)\rangle + (\langle \partial_\eta \Psi(t)|\Psi(t)\rangle)^2)\,.$$

While the time-crystal phase features a **many-body enhanced** sensitivity witnessed by a superlinear scaling of the QFI with system size the **optimal measurement** capable of saturating this fundamental bound **might not be feasible**. Analytically, we showed that if conditions [1]

$$\text{(I)}\ \operatorname{Tr}\left[(\partial_\eta H)\rho(s)\right] = 0 \quad \forall s \leq t,$$
$$\text{(II)}\ \langle \psi_{\mathbf{i_t}} | \phi_{\mathbf{i_t}} \rangle \in \mathbb{R} \quad \forall\, \mathbf{i_t},$$

with $|\phi_{\mathbf{i_t}}\rangle = |\partial_\eta \tilde{\psi}_{\mathbf{i_t}}\rangle / \sqrt{\langle \tilde{\psi}_{\mathbf{i_t}} | \tilde{\psi}_{\mathbf{i_t}}\rangle}$ and $|\psi_{\mathbf{i_t}}\rangle = K_{i_M} \dots K_{i_1} |\psi(0)\rangle$ are fulfilled photocounting and homodyne detection are **optimal** and the **QFI is saturated by the FI** associated with $\boldsymbol{i_t}$

$$\lim_{t\to\infty} \left(\frac{\mathcal{F}_{\text{SE}}(\omega,t) - \mathcal{F}_{\text{cl}}^{(p),(h)}(\omega,t)}{t} \right) = 0\,.$$

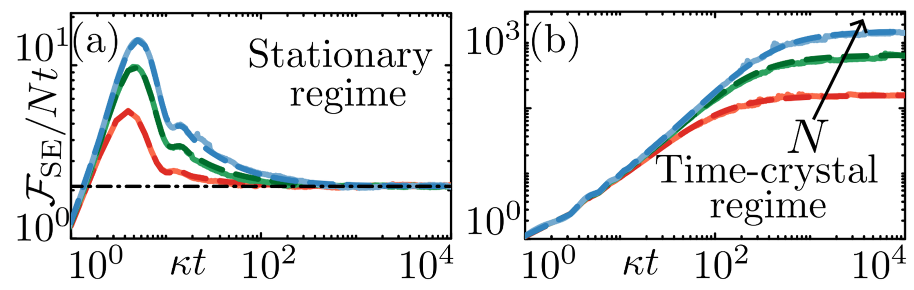

Fig. 3. **Saturating enhanced QFI in the time-crystal phase via photon counting.** System of *N* resonantly driven qubits, interacting with a (lossy) bosonic mode via Tavis-Cummings coupling.

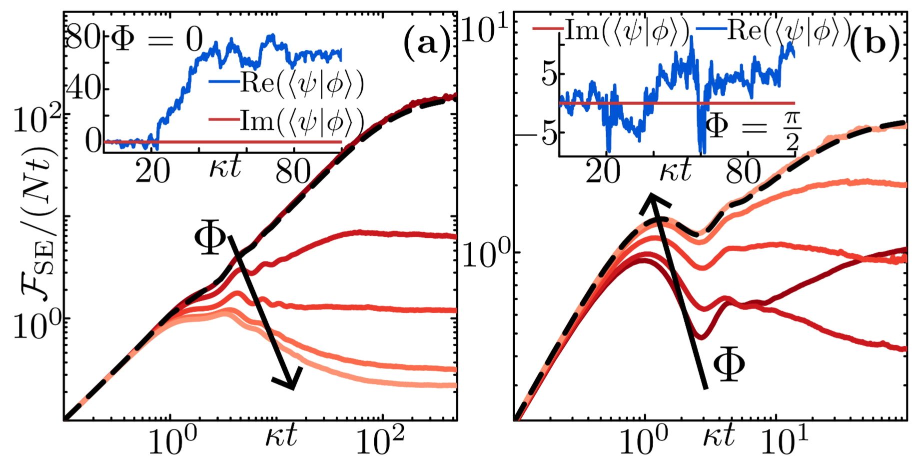

Fig. 4. **Saturating the QFI with homodyne detection.** System of *N* resonantly driven qubits, interacting with a (lossy) bosonic mode via Tavis-Cummings (a) or generalized Dicke coupling (b), with Φ being the phase used for homodyne detection.

With these easy-to-check criteria one can identify a class of models, where these continuous measurement schemes are optimal for sensing Ω. In the same way detrimental terms can be identified:

Model	Boundary time-crystal [2]	Driven qubit [3]	Driven Three-level system
Saturating states	$\sum_{M_z=-S}^{S} C_{M_z} \|S, M_z\rangle$ $C_{M_z}^* C_{M_z+1} \in \mathbb{I}$	$c_g \|g\rangle + c_e \|e\rangle$ $c_g^* c_e \in \mathbb{I}$	$c_g \|g\rangle + c_e \|e\rangle + c_d \|d\rangle$ $c_g \in \mathbb{I}(\mathbb{R}), c_e, c_d \in \mathbb{R}(\mathbb{I})$
Phase Φ (Homodyne)	$\pi/2$	$\pi/2$	$\pi/2$
Detrimental terms	ΔS_z	$\Delta \sigma_z$	$\Delta_e \|e\rangle\langle e\|$

4 Nonequilibrium phases for time-keeping

Spontaneous time-translation symmetry breaking, as apparent in **TC's**, is a possible resource for quantum clocks with **enhanced performance** quantified by resolution $\mathcal{R} = [\langle \mathcal{T} \rangle]^{-1}$ and accuracy $\mathcal{A} = \langle \mathcal{T} \rangle^2 / \text{Var}[\mathcal{T}]$. Here, $\mathcal{T}$ corresponds to the minimum time satisfying $N(\mathcal{T}) = M$, defining a "click" for a predefined threshold value M and thus can be interpreted as a **first passage time** [4].

Complex clock works $N(\mathcal{T})$ can feature an **enhanced tradeoff** between resolution and accuracy in the TC phase based on the **built-up of correlations** in time. Further, the accuracy scales as $\mathcal{A} \sim \sqrt{N}$ in the TC regime hinting towards a **many-body enhancement**.

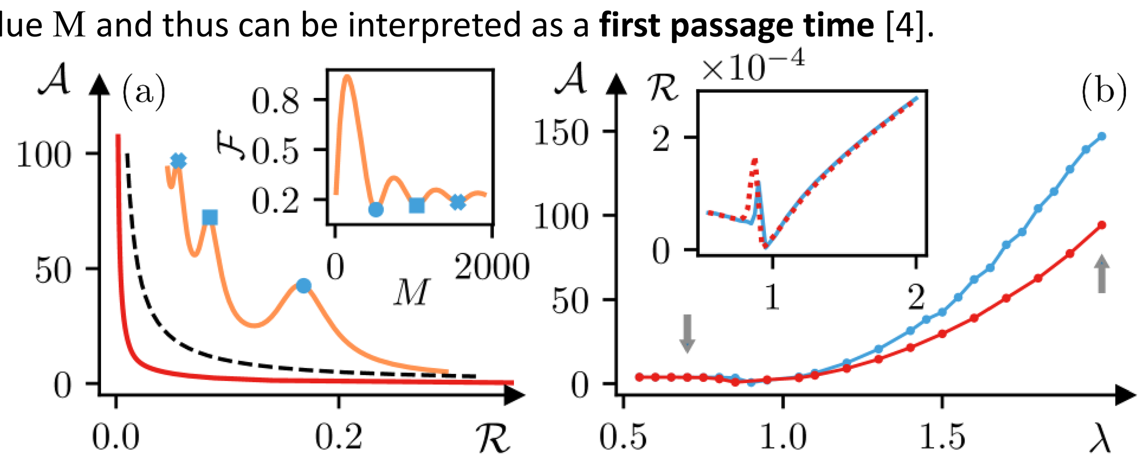

Fig. 5. **Enhanced accuracy-resolution tradeoff.** (a) Accuracy-resolution tradeoff below the critical value (red), above (orange) and Poissonian benchmark (dashed) for $N = 100$. (b) Accuracy (resolution) near the critical value $\lambda_c = 1$ for $N = 50{,}100$.

5 Further results and outlook

In the context of CoQuaDis we derived results, towards

- An **effective description** of dynamical many-body quantum systems [5,6], **quantum reservoir computing** based on spin-boson models [7], and **adiabatically driven** spin systems [8].

Future research will involve

- The exploration of the **link** between reservoir computing and parameter estimation, the **realization** of continuous monitoring in a trapped ion system and the **application of large deviation theory** to quantum clocks.

References

[1] R. Mattes, A. Cabot, F. Carollo, I. Lesanovsky, arXiv:2505.08756 (accepted in Phys. Rev. Lett.)
[2] F. Iemini, A. Russomanno, J. Keeling, M. Schirò, M. Dalmonte and R. Fazio, Phys. Rev. Lett. **121**, 035301 (2018)
[3] A. H. Kiilerich, K. Mølmer, Phys. Rev. A **89**, 052110 (2014) and Phys. Rev. A **94**, 032103 (2016)
[4] L. Viotti, M. Huber, R. Fazio, G. Manzano, arXiv:2505.08276
[5] R. Mattes, I. Lesanovsky, F. Carollo, Phys. Rev. Lett **134**, 070402 (2025)
[6] F. Carollo, I. Lesanovsky, Phys. Rev. Lett **133**, 150401 (2024)
[7] S. Das, G. L. Giorgi, R. Zambrini, arXiv:2510.00171
[8] P. J. Paulino, S. Teufel, F. Carollo, I. Lesanovsky, arXiv:2509.12075

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